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FRACTURE STUDY OF WINDSHIELD GLASS PANES

Final Report

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ABSTRACT

The major stresses to cause crack propagation in windshield glass panes are induced by bending moment which is resulted from the pressure differentials across the panes. Hence the stress intensity factors for finite plate with semi-elliptical surface flaw and edge crack under bending moments are examined. The results show that the crack growth will be upperbound if it is computed by using the stress intensity factor for finite plate with edge crack subjected to pure bending moments. Furthermore, if the ratio of crack depth to plate thickness, a/t , is within 0.3, the stress intensity factor can be conservatively assumed to be constant of being the value at a/t equal to zero. A simplified equation to predict structural life of glass panes is derived based on constant stress intensity factor. The accuracy of structural life is mainly dependent on how close the empirical parameter, m , can be estimated.

Introduction

Most structural materials contain certain surface flaw which is either inherent in the basic materials or is introduced during manufacturing, assembling or transporting processes. The surface flaw in glass panes is one of the major factors that affect the glass strength[1]**. The flaw depth of the glass is controlled by the way how the surface is polished. Its depth is usually at least three times of the diameter of the grinding particles used in surfacing finishing operation[2]. If the grinding techniques can be improved to a degree so that no surface flaw exists, the strength of the glass can reach as high as 2,000,000 psi. Without any doubt, the structural life of glass pane can be prolonged by reducing the surface flaws. Nevertheless, today's most sophisticated grinding technology can not diminish the flaw depth beyond 0.001 inch. The inner glass panes of flight vehicles are constantly subjected to pressure differentials across the panes. Consequently, the incipient flaw existing in the glass panes will start to propagate because of the flexural stresses inside the glass induced by pressures. The elastic theory of fracture mechanics has been widely utilized to investigate the induced stress in the crack zone and the behaviour of crack propagation of the glasses. Since the states of elastic stress and strain in the vicinity of the crack are characterized by the stress-intensity factors, extensive research in this area has been conducted in the past two decades. Exact solutions as well as approximate solutions for various flaw shapes in infinite as well as finite bodies have been obtained[3,4].

** Numerals in the brackets refer to the list of references.

The objectives of this study are to investigate the stress-intensity factors for analyzing the crack propagation of glass panes and to seek the efficient way of predicting the structural life of glass panes under service loads.

Stress-Intensity Factors

Fracture mechanics undoubtedly has been accepted by most design engineer as a major tool to prevent the structural system from brittle failure. Since imperfection exists in most engineering materials, attention has been concentrated on analyzing the stress redistribution in the small region of flaw to ensure the material toughness in flaw region is strong enough to avoid failure. This toughness is characterized as fracture toughness K_{Ic} , which is a constant for given materials. The crack size corresponding to fracture toughness for the specified stress is termed as critical crack size. In case the crack size passes beyond the critical threshold, crack propagation becomes unstable and fracture occurs. The stress-intensity factor was introduced to describe the stress behaviour in the neighborhood of crack tip for the crack size smaller than critical size. Therefore the main subject of fracture mechanics is to obtain the solution of stress-intensity factors for different problems.

In 1921, Griffith[5] uses the energy approach to establish the basic equation of stress-intensity factor for an infinite cracked plate sheet with a central crack subjected to a remote uniform tensile stress which is perpendicular to the crack. Griffith's formula only applies to infinite bodies. Errors may arise due to the finite size of crack solid as well as the plasticity

effect. Consequently, considerable efforts have been extended to investigate the stress-intensity factors for finite solids. Because of the complexities of the problems, the exact solutions of stress-intensity factors for various crack configurations in finite bodies are not available. Numerical techniques, such as finite element, boundary collocation, mapping, integral transform, and asymptotic expansion, etc. have been extensively used by many researchers to seek the approximate solutions for different cases. Shih[3] and rooke[4] summarize the stress-intensity factors for various configurations in great details.

Since the semi-elliptical surface flaw has been recognized as a closed approximation to natural flaw and since the major force to drive the crack propagation in glass panes is flexural stresses, it is of much interest to assess the effectiveness of the various stress-intensity factors of elliptical surface flaw in measuring crack growth in glasses.

The stress-intensity factors for a shallow, semi-elliptical surface flaw in a plate with finite thickness under uniform tension was developed by Irwin[6] basing upon Green and Sneddon [7] solution. Method of superposition and iteration has been used to find the solution for semi-circular surface flaw in half space under tension, bending load or other loading combination[8,9]. Smith[10] obtained an approximate solution for a semi-elliptical surface flaw in a plate with finite thickness due to tension and bending moment. Finite element method[11,12,13,14] and boundary-integral equation[15] also have been applied to such complex problems of two or three dimensional solids. Since no exact solution exists, experimental studies were conducted to assess

the accuracy of the approximate solution derived from numerical techniques[16,17,18].

The solutions of stress-intensity factors of the edge crack in a plate with finite thickness were investigated by Gross et.al.[19,20] using boundary collocation method. Their solutions agree well with the experimental data[21,22].

The solution of stress-intensity factor of semi-elliptical surface flaw in a plate with finite width can be expressed in the following general form;

$$K_I = M S / \pi a / Q \quad \text{----- (1)}$$

$$\text{where } Q = \phi^2 - 0.212 (S_o^2 / S_y^2)$$

$$\text{and } \phi = \int_0^{\pi/2} \{ \cos \theta + (a/c) \sin \theta \} d\theta$$

K_I is the stress intensity factor; S is the maximum flexural stress at the outer fiber; S_y is the yield strength and a and c are the minor and major axes of the ellipse, respectively. M is a dimensionless magnification factor, dependent upon the crack geometry, crack depth, plate thickness and stress location. The second term in function Q is the correction factor for plasticity effect and will be neglected in this study since the glass is a brittle material. Q is equal to one for edge crack as a/c approaches zero and becomes $\pi/2$ for semi-circular crack as $a = c$. In order to visualize the effect of parameters on the value of magnification factor M , Smith's[10], Marrs'[17] and Gross'[20,22] solutions at the point of maximum crack depth for surface flaw are plotted in Fig. 1 as function of a/c and a/t . When the ratio a/c approaches zero, Smith's solution closely agrees with Gross'

solution, which can be expressed in terms of power series as follows;

$$M = 1.12 - 1.39(a/t) + 7.32(a/t)^2 - 13.1(a/t)^3 + 14(a/t)^4 \quad \text{---(2a)}$$

The assumptions used to derive equation 2a are (1) finite thickness plate with central edge crack subjected to pure bending moments; (2) the direction of moment is perpendicular to crack plane; and (3) no surface traction exists on both front and back surfaces. Also Equation 2a is only valid for the ratio of crack depth to plate thickness not greater than 0.6. Within this limit, the error by using Equation 2a is less than 1%. In case shear force and bending moment exert simultaneously to the plate, the stress intensity factors are influenced by the ratio of plate span to plate thickness. The magnification factors calculated by Gross[22] for a simply supported plate with a span of L and edge crack in the central under central loads for $L/t=8$ and $L/t=4$ are shown in Equations 2b and 2c and plotted in Fig. 2.

$$M = 1.11 - 1.55(a/t)^2 + 7.71(a/t)^3 - 13.5(a/t)^4 + 14.2(a/t)^4 \quad \text{---(2b)}$$

for $L/t = 8$

$$M = 1.09 - 1.73(a/t)^2 + 8.20(a/t)^3 - 14.2(a/t)^4 + 14.6(a/t)^4 \quad \text{---(2c)}$$

for $L/t = 4$

As the span is lengthened, the effect of shear force becomes negligible. Hence the magnification factor for the three-point moment approaches to that of pure bending as it is illustrated in Fig. 2. Fig. 1 or 2 explains that the magnification factors decrease, at first, as a/t increases. Then they reach the minimum where a/t is approximately equal to 0.13 for pure bending and $a/t=0.14$ and 0.15 for $L/t=8$ and 4, respectively, for three point moment. Thereafter, both the magnification factors and a/t increase

simultaneously. Conclusion can be drawn from Figs. 1 and 2 that the stress-intensity factor of edge crack in a plate with finite thickness subjected to pure bending loads is more critical than that of semi-elliptical surface flaw under same conditions and is also more severe than that of edge crack under three-point moment. Therefore, to study the crack growth and to predict the structural life of glass panes, the stress-intensity factor of edge crack under pure bending moments should be considered to warrant the solutions in the safe domain.

Crack Growth

According to the experimental study done by Wiederhorn et.al. [23 to 28] the crack velocity of flaw propagation in glass is a function of stress-intensity factor, K_I , and the environments. In this study, the effect of the environments on crack velocity will be neglected. Consequently, the relationship between crack velocity and the stress-intensity factor can be expressed either in power function

$$V = D K_I^m \quad \dots \quad (3a)$$

or in exponential form

$$V = V_0 \exp[B K_I] \quad \dots \quad (3b)$$

where D , V_0 , B and m are empirical constants. These constants can be determined from experimental data collected by conducting double-cantilever-beam experiments. If the flaw depth at any time, T , is denoted by a , then the crack velocity is defined as follows:

$$V = \frac{da}{dT} \quad \dots \quad (4a)$$

or

$$da = V dT \quad \dots \quad (4b)$$

Substitute Equations 1 and 2a into Equations 4b, we obtain

$$\int_{a_0}^{a_e} M a^{-m} (-m/2) da = \int_0^T (D S / \pi)^m dT \quad \text{----- (5a)}$$

or

$$\int_{a_0}^{a_e} \text{Exp}[B M S / \pi a] da = \int_0^T V dT \quad \text{----- (5b)}$$

Since the magnification factor is not a linear function of flaw depth, no explicit expression for crack growth can be obtained.

However, Equations 5a or 5b can be solved by numerical method.

In order to view the effect of parameter, a/t , on the crack propagation, a load spectrum shown in Figs. 4 is utilized to compute the crack growth from Equation 5b. The results show that the total crack growth mainly depends on stress intensity factor. As stress intensity factor decreases, the crack growth slows down. The growth rate approaches the minimum when stress intensity factor reaches the smallest value. Then the growth rate accelerates as stress intensity factor increases.

To study the fracture behaviour of the windshield glass for shuttle orbiter, Hayashida et.al. [32] used $M=1.12$, which corresponding to M value by setting $a/t = 0$ in Equation 2a. According to the graphic indication from Fig. 5, the calculated crack growth with $M = 1.12$ exceeds the actual crack growth by considering M as function of a/t as long as a/t is less than 0.3. Accordingly, if M is considered to be 1.12 for a/t being less than 0.3, the crack growth will be the upper bound. If M is assumed to be constant, Equation 5a can be integrated.

$$a_j = \{ a_{j-1}^{(1-m/2)} + (1-m/2) D \left(S_j M_j / \pi \right)^m \Delta T_j \}^{(1-m/2)} \quad \text{----- (6)}$$

where S_j is the applied stress at time interval ΔT_j and a_j

is the total crack depth at time T . In the case, the sustained stress is constant, i.e. $S_j = S$ for all j , Equation 6 becomes

$$a_f = \{ a_0 + (1-m/2) D (S^m / \pi) T \}^{(1-m/2)} \quad \text{--- (7)}$$

a_f is the total crack depth and T is the time duration that the sustained stress is acting upon the object.

Structural Life of Glass Panes

To study the structural life of glass panes, it is assumed that the magnification factor, M , be constant. It is justified since the variation of a/t during the crack propagation is very small. Wiederhorn et.al.[24,25] have derived the expression for predicting the structural life of glass for skylab windows by using Equations 3b and 4a.

$$T = \frac{2 \exp[-BSM/\pi a_c] (1+BSM/\pi a_c)}{(BSM/\pi)^2 V} \left(\frac{\sqrt{a_c}}{1 - \exp[-BSM/\pi a_c] \left(\frac{\sqrt{a_c}}{a_0} - 1 \right)} \right)^{\frac{(1+BSM/\pi a_c)}{(1+BSM/\pi a_c)}} \quad \text{--- (8)}$$

a_c is the critical flaw depth which can be determined from the fracture toughness, K_c ;

$$a_c = \left\{ \frac{K_c^2}{(SM/\pi)^2} \right\}^{1/2} \quad \text{--- (9)}$$

Since, in practice, a_c is usually much larger than the initial flaw depth, a_0 , after discarding small terms, Equation 8 becomes

$$T = \frac{2 \exp[-BSM/\pi a_0] / \sqrt{a_0}}{BSM / \pi V} \quad \text{--- (10)}$$

Equation 10 can be used to estimate the structural life of glass pane if the material constants of the particular glass are known. If Equation 10 is expressed in terms of stress-intensity factor and crack velocity, it becomes

$$T = \frac{2 a_o}{V_B K} \quad \dots \quad (11)$$

$$1 \quad I1$$

V_1 is the initial crack velocity corresponding to K_{I1} .

This equation can be used to estimate the service life of glass panes if the information of the similar glass panes is given. This information can be obtained from experimental work. The service lives of the glass panes are related as follows:

$$T = \frac{V_1 K_{I1}}{2 V_2 K_{I2}} T \quad \dots \quad (12)$$

$$2 \quad 1$$

Wiederhorn [25] has shown that the results computed from Equation 9 agree with his experimental data.

Since, as mentioned previously, the crack velocity can also be expressed in power formation, the time for flaw depth to reach the critical point is derived from Equations 3a and 4a.

$$T = \frac{2 a_o}{D(S M / \pi a_o)^m (m-2)} \left\{ 1 - \left(\frac{a_o}{a_c} \right)^{m/2-1} \right\} \quad \dots \quad (13)$$

The last term in the right-hand side of Equation 13 can be omitted, since a_c is usually much larger than a_o .
Equation 13 can be simplified as shown in the following:

$$T = \frac{2 a_o}{D(S M / \pi a_o)^m (m-2)} \quad \dots \quad (14)$$

It is to be noted that if the ratio, a_o / a_c , is not very small, the error of T calculated from Equation 11 will mainly rely on m . The larger m is, the smaller the error will be. For example, if a_o / a_c is equal to 0.5, the percentages of error of T are about 0.8% and 0.006% with m being 16 and 30,

respectively. After multiplying both sides of Equation 14 by S^m , take logarithm on both sides.

$$\ln(T) + m \ln(S) = \ln(C) \quad \dots \quad (15)$$

where $C = \frac{2 a_0}{D(M / \pi a_0)^m (m-2)}$ (16)

If they are plotted in log-log scale as shown in Fig. 6, it is a straight line with a slope of $-1/m$ for time, T , and stress, S .

Because the constant, C , is a function of the initial flaw depth, the lines for various a_0 should parallel to one another.

From Figure 6, the approximate life of glass under any stress level can easily be determined. The accuracy of the results will mainly be controlled by empirical constants, such as D and m .

In order to predict the service life of glass by using Equation 16, it is necessary to measure the initial flaw depth, in addition to the determination of empirical constants D and m . However, the relationship of the service lives of two identical glass panes in the same environment can be formulated from Equation 16.

$$T_2 = \left(\frac{S_1}{S_2} \right)^{\frac{1}{m}} \left(\frac{a_{01}}{a_{02}} \right)^{\frac{m}{m-2}} T_1 \quad \dots \quad (17)$$

If the initial flaw depths for both glass panes are the same, then Equation 17 becomes

$$T_2 = \left(\frac{S_1}{S_2} \right)^{\frac{1}{m}} T_1 \quad \dots \quad (18)$$

In supporting the validity of Equation 12, Wiederhorn[25]

obtained the following data from his experimental study.

$$\begin{aligned} S_1 &= 8.4 \text{ MN/m}^2, V_1 = 1.4 \times 10^{-4} \text{ m/s}, T_1 = 5 \text{ sec} \\ K_{I1} &= 0.7 \text{ MN/m}^{3/2} \\ S_2 &= 4.2 \text{ MN/m}^2, V_2 = 2 \times 10^{-9} \text{ m/s}, K_{I2} = 0.35 \text{ MN/m}^{5/2} \end{aligned}$$

By using Equation 12, T_2 is equal to 7×10^{-5} sec, which confirms with his experimental data. Though m is not available from his study, it can be estimated by substituting all V 's and K 's into Equation 3a. We obtain

$$m = \frac{\ln(V_1/V_2)}{\ln(K_{I1}/K_{I2})} = 16.095 \quad \text{---(19)}$$

By substituting m into Equation 18, T_2 is approximately equal to 3.5×10^{-5} sec. It does not agree well with the experimental results due to the lack of accurate information of constant m . From Wiederhorn's study[26] of crack velocity for various silica glasses, he shows that the constant m is in the range of the 30's. Since the service time calculated from Equation 18 is very sensitive to the constant m , extensive experimental study on m is a must. If accuracy of m is reachable, the service life of glass pane computed from Equation 14 will be very accurate for large cracks. Even for a small crack, Equation 18 is still a good estimator.

Conclusions

The driving forces to initiate the propagation of surface flaw in the inner glass panes of flight vehicles are mainly due to flexural stresses induced by the pressure differentials

across the panes. Hence the stress intensity factors for various surface flaw shapes in a finite thickness plate subjected to bending loads are examined. The results show that the solutions obtained by using the stress intensity factor for plate with edge crack under pure bending moment to analyze the fracture of glass panes are more conservative than that of semi-elliptical surface flaw under bending loads or edge crack under three point moment. If the ratio of the crack depth to plate thickness is within 0.3, the solution by using magnification factor equal to 1.12 will be the upper bound for crack growth and lower bound for structural life. The design based on this assumption of M being equal to 1.12 is conservative.

A simple equation (Equation 18) is obtained to predict the structural life of one glass pane based on the information of the other. Since there is no experimental data available to measure the effectiveness of Equation 18 and the result obtained from this equation is very sensitive to the value of m , it is recommended that an extensive experiment should be conducted to study the variations of parameters as well as the verification of Equation 18.

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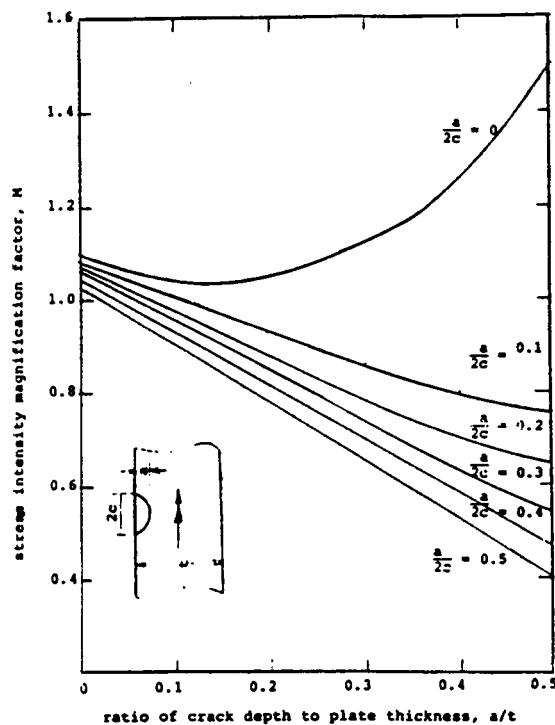


Figure 1. Stress Intensity Magnification Factor for
Semi-Elliptical Surface Flaw at Maximum Depth

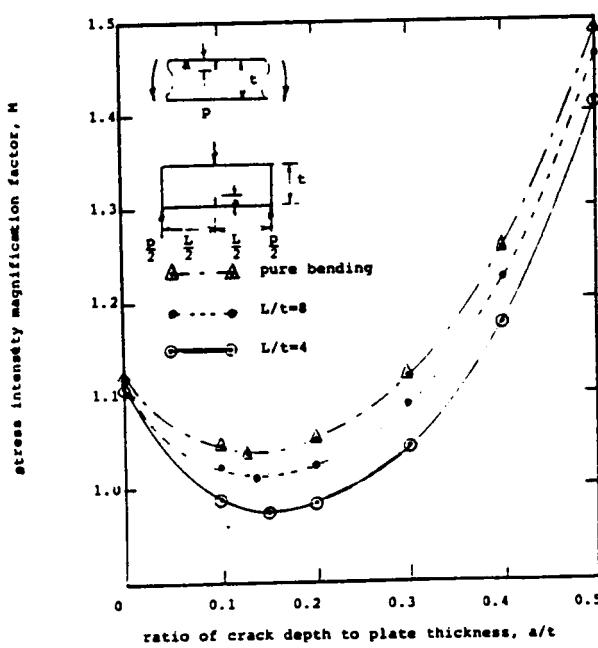


Figure 2. Stress Intensity Magnification Factor for
Edged Crack Plate Under Bending Moments

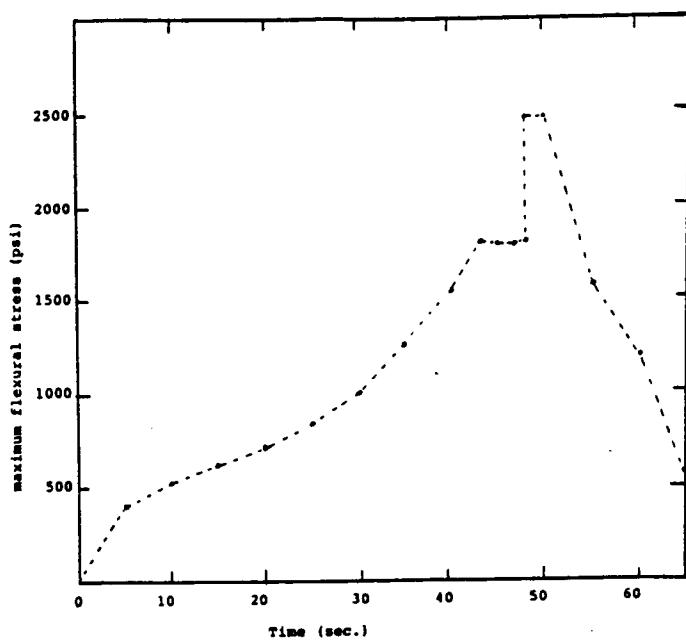


Figure 3. Load Spectrum

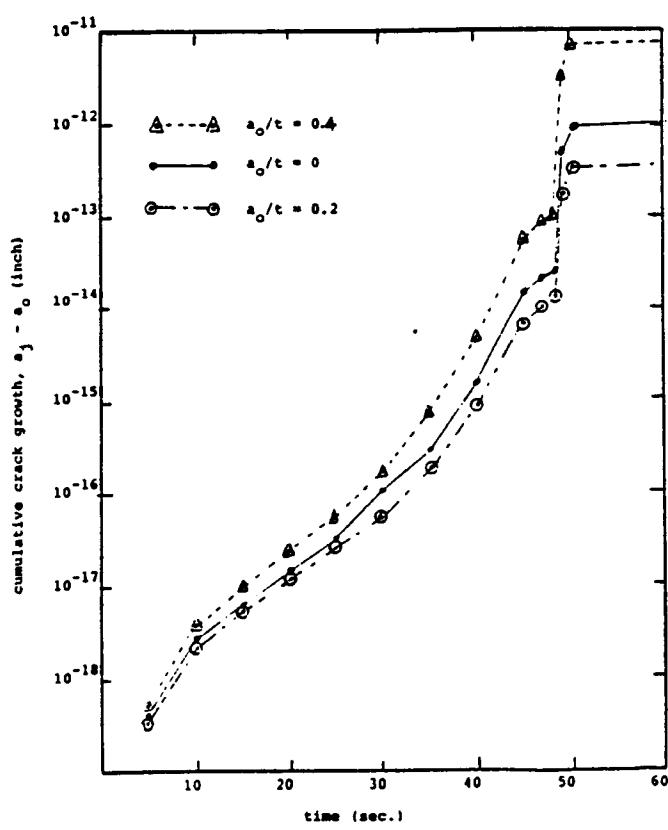


Figure 4. Cumulative Crack Growth

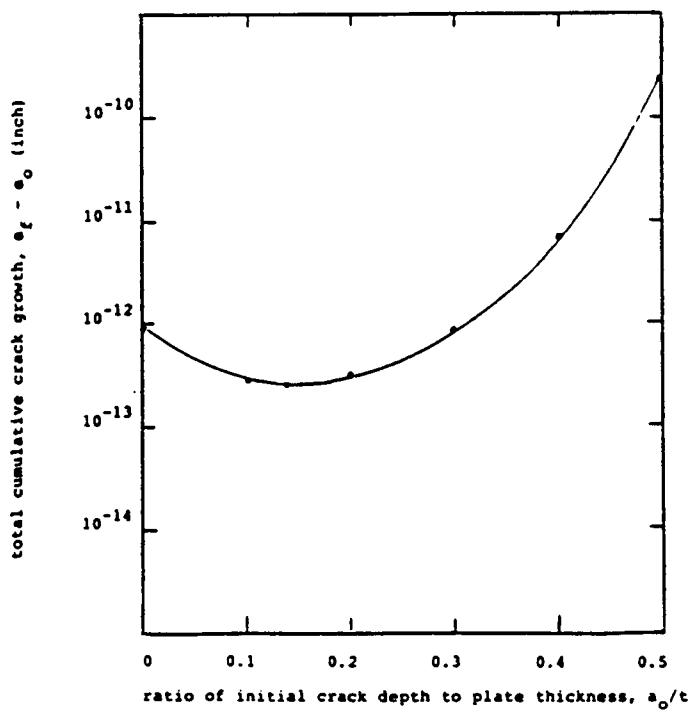


Figure 5. Total Cumulative Crack Growth

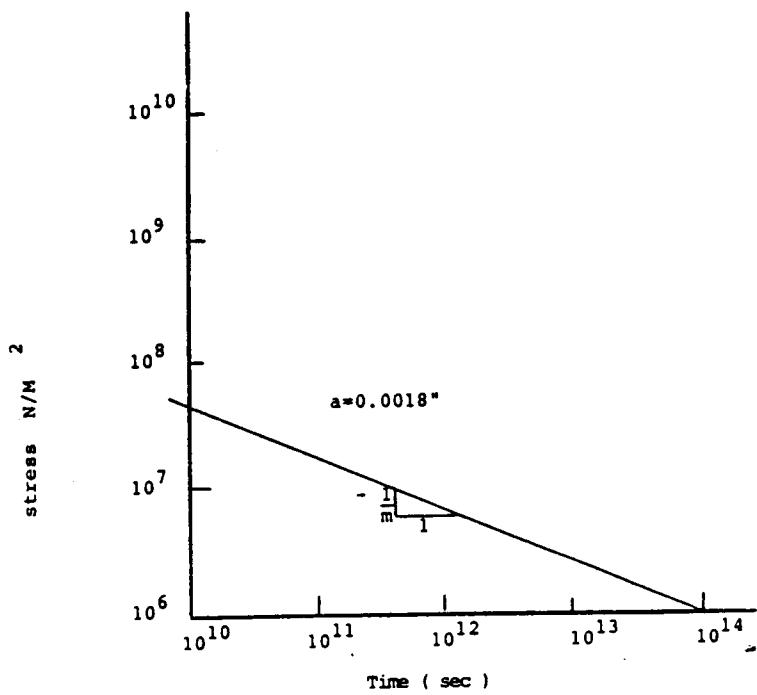


Figure 6. Stress-Time Diagram